BUILDING A FIBONACCI PHYLLOTAXIS ARRAY ON A SPREADSHEET

Spreadsheet “growth” algorithm for Fibonacci (21,34)-phyllotaxis

1) Construct a blank spreadsheet comprised of 21 rows x 34 columns.

(2) Place the number ‘1’ in [*row*, *col*] lattice position [21, 34].

(3) Place the number ‘2’ in the position [row minus 8, column minus 13], which is [*row*, *col*] value [13, 21].

(4) Place the number ‘3’ (i.e., the third floret added to the matrix, designated by the color cyan) in the lattice position of [row minus 8, column minus 13], which is equal to [*row* , *col*] value [5, 8].

(5) Place the number ‘4’ (i.e., the fourth floret added to the matrix, designated by the color blue) in the lattice position [row minus 8, column minus 13], which is [*row*, *col*] value [18, 29]. Note how the lattice actually forms a cylinder and the algorithm wraps around from left to right indicated by the arrow to the right of the number 4.

(6) Place the number ‘5’ (i.e., the fifth floret added to the matrix, designated by the color purple) in the position [row minus 8, column minus 13], which is [*row*, *col*] value [10, 16].

Note that this algorithm creates a sloping, saw tooth array of numbers. As will be demonstrated, this type of sloping, Fibonacci phyllotaxis matrix is a highly efficient and elegant prime number sieve.

21	694	673	652	631	610	589	568	547	526	505	484	463	442	421	400	379	358	337	316	295	274	253	232	211	190	169	148	127	106	85	64	43	22	1
20	660	639	618	597	576	555	534	513	492	471	450	429	408	387	366	345	324	303	282	261	240	219	198	177	156	135	114	93	72	51	30	9		
19	626	605	584	563	542	521	500	479	458	437	416	395	374	353	332	311	290	269	248	227	206	185	164	143	122	101	80	59	38	17				
18	592	571	550	529	508	487	466	445	424	403	382	361	340	319	298	277	256	235	214	193	172	151	130	109	88	67	46	25	4					
17	558	537	516	495	474	453	432	411	390	369	348	327	306	285	264	243	222	201	180	159	138	117	96	75	54	33	12							
16	524	503	482	461	440	419	398	377	356	335	314	293	272	251	230	209	188	167	146	125	104	83	62	41	20									
15	490	469	448	427	406	385	364	343	322	301	280	259	238	217	196	175	154	133	112	91	70	49	28	7										
14	456	435	414	393	372	351	330	309	288	267	246	225	204	183	162	141	120	99	78	57	36	15												
13	422	401	380	359	338	317	296	275	254	233	212	191	170	149	128	107	86	65	44	23	2													
12	388	367	346	325	304	283	262	241	220	199	178	157	136	115	94	73	52	31	10															
11	354	333	312	291	270	249	228	207	186	165	144	123	102	81	60	39	18																	
10	320	299	278	257	236	215	194	173	152	131	110	89	68	47	26	5																		
9	286	265	244	223	202	181	160	139	118	97	76	55	34	13																				
8	252	231	210	189	168	147	126	105	84	63	42	21																						
7	218	197	176	155	134	113	92	71	50	29	8																							
6	184	163	142	121	100	79	58	37	16																									
5	150	129	108	87	66	45	24	3																										
4	116	95	74	53	32	11																												
3	82	61	40	19																														
2	48	27	6																															
1	14																																	
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**

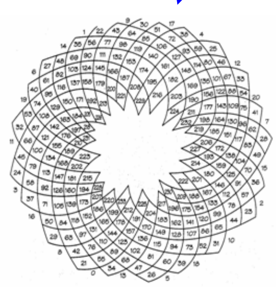
This sloping array of numbers defines the surface or skin of a *semi-infinite cylinder*. Column # 34 (right) when connected with column #1 (left) forms the cylinder, and the arrows wrap around it.

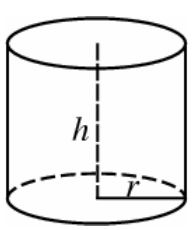
ABOVE: 21x34 initial daisy growth lattice. The growth algorithm determines the spatial position of successive daisy florets in the lattice, shown in two dimensions on a spreadsheet. Phyllotaxis integer lattice in a 2-dimensional spreadsheet, showing the algorithm that expresses 137.5-degree divergence angle the first 5 florets on a plane representing a semi-infinite cylinder.

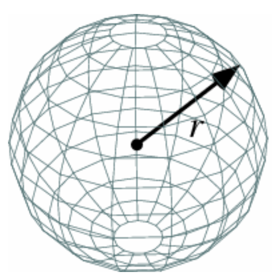
27																																	226	05
26																																213	19	
25																														221	200	179	158	37
24																												229	208	187	166	145	124	103
23																											216	195	174	153	132	111	90	69
22																									224	203	182	161	140	119	98	77		
21																							232	211	190	169	148	127	106	85	64	43	22	1
20																						219	198	177	156	135	114	93	72	51	30	9		
19																				227	206	185	164	143	122	101	80	59	38	17				
18																			214	193	172	151	130	109	88	67	46	25	4					
17																	222	201	180	159	138	117	96	75	54	33	12							
16															230	209	188	167	146	125	104	83	62	41	20									
15														217	196	175	154	133	112	91	70	49	28	7										
14												225	204	183	162	141	120	99	78	57	36	15												
13										233	212	191	170	149	128	107	86	65	44	23	2													
12									220	199	178	157	136	115	94	73	52	31	10															
11							228	207	186	165	144	123	102	81	60	39	18																	
10						215	194	173	152	131	110	89	68	47	26	5																		
9				223	202	181	160	139	118	97	76	55	34	13																				
8		231	210	189	168	147	126	105	84	63	42	21																						
7	2	197	176	155	134	113	92	71	50	29	8																							
		163	142	121	100	79	58	37	16																									
5	150	129	108	87	66	45	24	3																										
4	116	95	74	53	32	11																												
	82	61	40	19																														
		7	6																															
1	14																																	
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34

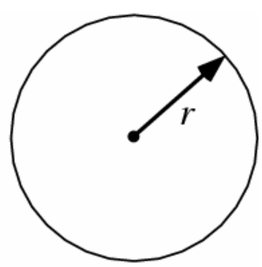
✄ Use a pair of scissors to cut along the dotted lines. Join the left and right ends of the matrix (columns #1 and #34, respectively) to form a cylinder or other shapes, which are possible with a stretchable spreadsheet substance such as rubber.

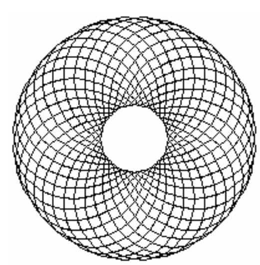
daisy flower as a spreadsheet











FROM LEFT TO RIGHT: A numbered daisy inflorescence, a cylinder, a sphere, a disk and a torus: all adapted by a phyllotaxis growth lattices. A daisy appears to correspond to a disk, flattened cone, or a torus.

Daisy integer lattice on a spreadsheet, showing the first 233 numbers added to the matrix. To "grow" the matrix, simply add "34" to the number at the top of each column.

27																																	226	205
26																																213	192	171
25																														221	200	179	158	137
24																												229	208	187	166	145	124	103
23																											216	195	174	153	132	111	90	69
22																									224	203	182	161	140	119	98	77	56	35
21																							232	211	190	169	148	127	106	85	64	43	22	1
20																						219	198	177	156	135	114	93	72	51	30	9		
19																				227	206	185	164	143	122	101	80	59	38	17				
18																			214	193	172	151	130	109	88	67	46	25	4					
17																	222	201	180	159	138	117	96	75	54	33	12							
16															230	209	188	167	146	125	104	83	62	41	20									
15														217	196	175	154	133	112	91	70	49	28	7										
14												225	204	183	162	141	120	99	78	57	36	15												
13										233	212	191	170	149	128	107	86	65	44	23	2													
12									220	199	178	157	136	115	94	73	52	31	10															
11							228	207	186	165	144	123	102	81	60	39	18																	
10						215	194	173	152	131	110	89	68	47	26	5																		
9				223	202	181	160	139	118	97	76	55	34	13																				
8		231	210	189	168	147	126	105	84	63	42	21																						
7	218	197	176	155	134	113	92	71	50	29	8																							
6	184	163	142	121	100	79	58	37	16																									
5	150	129	108	87	66	45	24	3																										
4	116	95	74	53	32	11																												
3	82	61	40	19																														
2	48	27	6																															
1	14																																	
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**

Successive numbers in each column of the matrix increase by (n + 34) in this direction.

This is an important figure. It shows the numerical structure of the daisy integer matrix is governed by Fibonacci differences among adjacent elements of the matrix. The black arrows indicate the direction of integer sequences in the matrix. The integer sequences have consecutive integers separated by a Fibonacci number.

21	694	673	652	631	610	589	568	547	526	505	484	463	442	421	[illegible]	379	358	337	316	295	274	253	232	211	190	169	148	127	106	85	64	43	22	1
20	660	639	618	597	576	555	534	[illegible]	492	471	450	429	408	387	[illegible]	345	324	303	282	261	240	219	198	177	156	135	114	93	72	51	30	9		
19	626	605	584	563	542	521	500	[illegible]	458	437	416	395	374	353	[illegible]	311	290	269	248	227	206	185	164	143	122	101	80	59	38	17				
18	592	571	550	529	508	487	466	[illegible]	424	403	382	361	340	319	298	277	256	235	214	193	172	151	[illegible]	[illegible]	88	67	46	25	4					
17	558	537	516	495	474	453	432	411	390	369	348	327	306	285	264	243	222	201	180	159	138	117	[illegible]	[illegible]	54	33	12							
16	524	503	482	461	440	419	398	377	356	335	314	293	272	251	230	209	188	167	146	125	104	[illegible]	[illegible]	41	20									
15	490	469	448	427	406	385	364	343	322	301	280	259	238	217	196	175	154	133	112	91	70	49	28	7										
14	456	435	414	393	372	351	330	309	288	267	246	225	204	183	162	141	120	99	78	57	36	15												
13	422	401	380	359	338	317	296	275	254	233	212	191	170	149	128	107	86	65	44	23	2													
12	388	367	346	325	304	283	262	241	220	199	178	157	136	115	94	73	52	31	10															
11	354	333	312	291	270	249	[illegible]	[illegible]	186	165	144	123	102	81	60	39	18																	
10	320	299	278	257	236	215	[illegible]	[illegible]	152	131	110	89	68	47	26	5																		
9	286	265	244	223	202	181	[illegible]	[illegible]	118	97	76	55	34	13																				
8	252	231	210	189	168	147	126	105	84	63	42	21																						
7	218	197	176	155	134	113	92	71	50	29	8																							
6	184	163	142	121	100	79	58	37	16																									
5	150	129	108	87	66	45	24	3																										
4	116	95	74	53	32	11																												
3	82	61	40	19																														
2	48	27	6																															
1	14																																	
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34

F34 F55 F13 F21

ABOVE: The four principal parastichies shown in a section of a semi-infinite cylinder represented by a 2-dimensional spreadsheet. Fibonacci number sequences determine the structure of the phyllotaxis growth lattice (daisy shown above)

For example, the four parastichies (or spiral sets of florets) which intersect at or emanate from the number 5 in the daisy lattice are comprised of the following four integer sequences:

***The F_{13} parastichy –** [e.g., (5, 18, 34, 44, 57, 70, 83, ...)]
Consecutive integers in the sequence are separated by the Fibonacci number, **13**

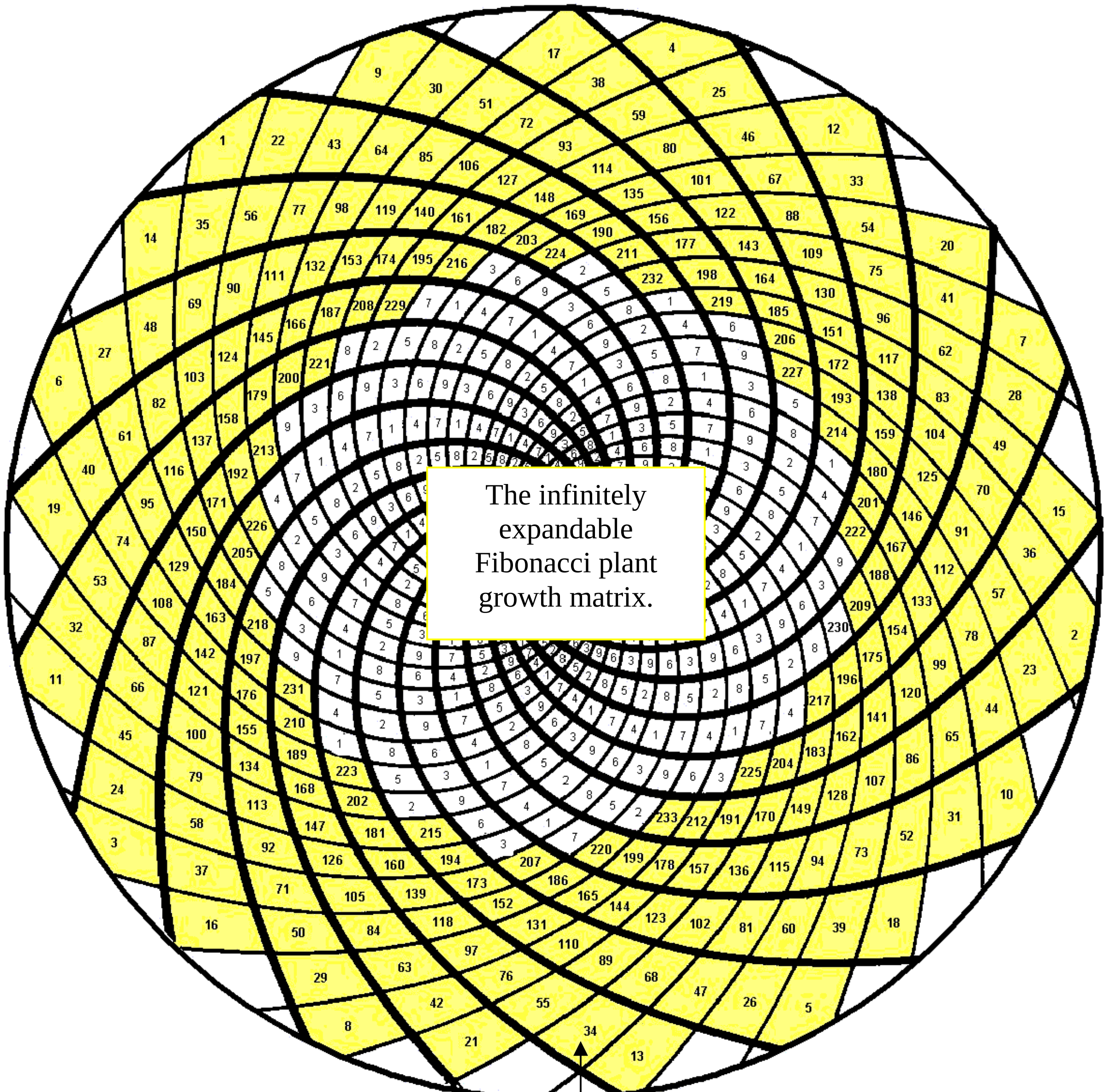
The F_{21} parastichy – [e.g., (5, 26, 47, 68, 89, 110, 131, ...)]
Consecutive integers in the sequence are separated by the Fibonacci number, **21**

The F_{34} parastichy – [e.g., (5, 39, 73, 107, 141, 175, 209, ...)]
Consecutive integers in the sequence are separated by the Fibonacci number, **34**

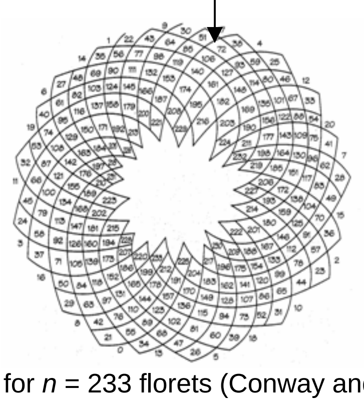
The F_{55} parastichy – [e.g., (5, 60, 115, 170, 225, 280, ..)]
Consecutive integers in the sequence are separated by the Fibonacci number, **55**

*F stands for 'Fibonacci'

Relationship of parastichy number series to Fibonacci numbers. The numbers (**13**, **21**, **34** and **55)** are consecutive numbers in the following Fibonacci sequence: [1, 1, 2, 3, 5, 8, **13, 21, 34, 55** ...]. These sequences define the entire growth process in terms of spatial and temporal location and occurrence of numbers (florets) in the matrix. The same Fibonacci differences are manifest at any point of origin within the matrix.



Simulated daisy integer matrix (above), expanded [after Conway and Guy (1996)]. The first 233 florets added to the lattice are highlighted in yellow. Florets toward the center of the lattice are numbered in modulo 9 and are shown in white. Numbers near the center of the matrix are represented in Base 10 (modulo 9) to conserve space. <u>NOTE</u>: If this matrix were expanded to a very large size, one could compare the pattern of prime numbers with the famous Ulam's Spiral. One may be able to observe the "organic" patterns of prime numbers heretofore unrecognized.

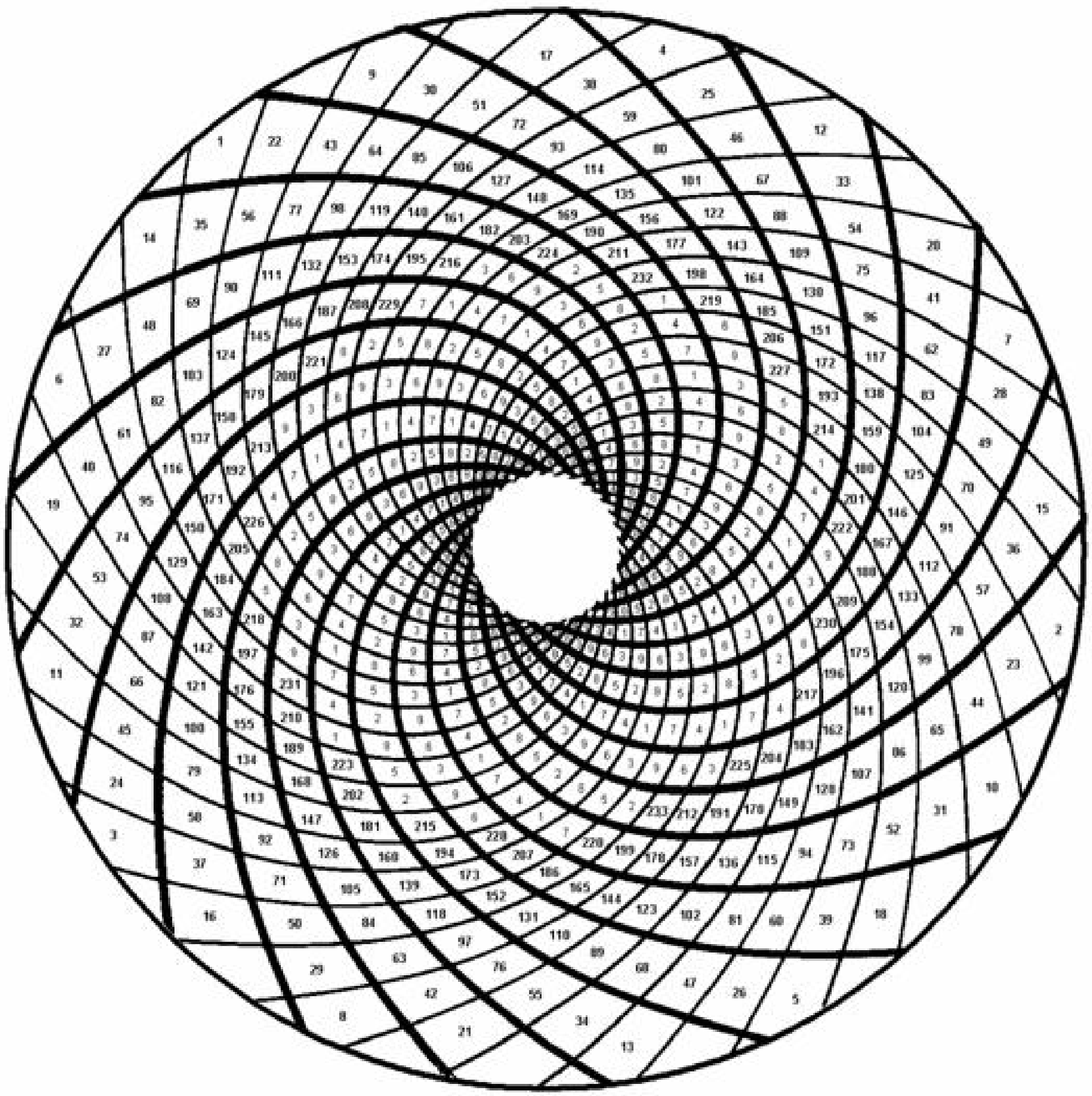


Daisy image for n = 233 florets (Conway and Guy, 1996)

THE 3-DIMENSIONAL BOTANICAL INTEGER LATTICE, *in situ.*
A DAISY INFLORESENCE WITH FIBONACCI (21, 34)-PHYLLOTAXIS.

A **botanical integer lattice** may be defined as a matrix of integers 1 to *n* used to map the spatial location of plant organs or units, such as florets in a daisy capitulum. As already mentioned, integers are assigned in arithmetic progression to new units as they appear in the plant lattice. Integer maps have been produced by phyllotaxis researchers for a wide range of plants. The maps were useful in developing several models and theories of phyllotaxis.

The first *n* = 233 floret nodes in the lattice were numbered in arithmetic progression with Fibonacci differences 21 and 34 for a **daisy** or **chrysanthemum** with (21, 34)-phyllotaxis (after Conway and Guy, 1996).



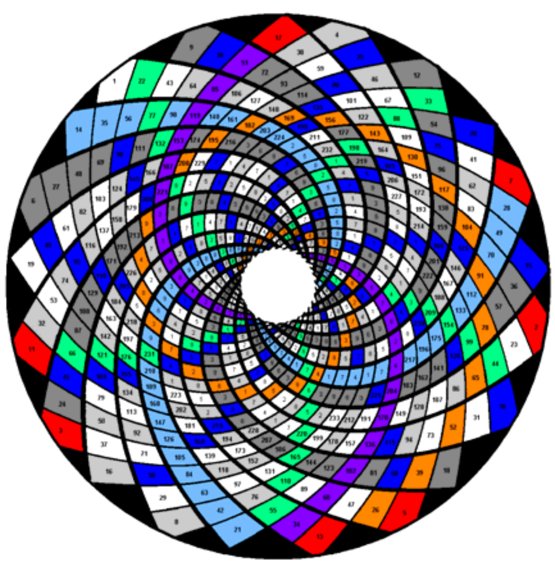
A botanical integer lattice for Fibonacci (21,34)-phyllotaxis , modeling florets in a daisy capitulum. Nodes are numbered in arithmetic progression for the first 233 florets (integers) added to the lattice. Florets nearer the center are numbered in modulo 9. Florets are numbered with Fibonacci differences 13, 21, 34 and 55 along the four principal parastichies, F_{13} , $F_{21,}$ F_{34} , and F_{55} .

DAISY PRIME NUMBER SIEVE

FOR *N* = 233 FLORETS

USING A SIMULATED DAISY FLOWER

(THREE DIMENSIONS)



The Sieve of Eratosthenes, an algorithm for making tables of primes (for comparison with phyllotaxis sieve).

Compared with the famous *Sieve of Eratosthenes*, the *Daisy Prime Sieve* functions by a similar algorithm. The main difference is that the number matrices vary significantly in comparison. The Eratosthenes matrix (sieve) is linear and numbers are placed in a linear, arbitrary array. The Phyllotaxis sieve (matrix) is not arbitrary; rather is it empirically derived, not random, systematic, and symmetrical and a completely natural product of Fibonacci numbers. A simple, stepwise version of the Sieve of Eratosthenes (for n = 646) is presented as follows:

(1) Sequentially write down the integers 1 to n. In this case, 1 to 64.

1	**2**	**3**	**4**	**5**	**6**
7	**8**	**9**	**10**	**11**	**12**
13	**14**	**15**	**16**	**17**	**18**
19	**20**	**21**	**22**	**23**	**24**
25	**26**	**27**	**28**	**29**	**30**
31	**32**	**33**	**34**	**35**	**36**
37	**38**	**39**	**40**	**41**	**42**
43	**44**	**45**	**46**	**47**	**48**
49	**50**	**51**	**52**	**53**	**54**
55	**56**	**57**	**58**	**59**	**60**
61	**62**	**63**	**64**		

(2) Find the first number in sequence after 1: the number 2. Circle the number 2, because it has not been crossed out yet and is therefore a prime number. Cross out all numbers > 2 which are divisible by two (i.e., every second number; red arrows).

1	2	**3**	**4**	**5**	**6**
7	**8**	**9**	**10**	**11**	**12**
13	**14**	**15**	**16**	**17**	**18**
19	**20**	**21**	**22**	**23**	**24**
25	**26**	**27**	**28**	**29**	**30**
31	**32**	**33**	**34**	**35**	**36**
37	**38**	**39**	**40**	**41**	**42**
43	**44**	**45**	**46**	**47**	**48**
49	**50**	**51**	**52**	**53**	**54**
55	**56**	**57**	**58**	**59**	**60**
61	**62**	**63**	**64**		

(3) Find the smallest remaining number > 2. It is 3. It has not been crossed out. Circle it and cross out all numbers > 3 which are divisible by three (every third number; blue arrows).

1	2	**3**	**4**	**5**	**6**
7	**8**	**9**	**10**	**11**	**12**
13	**14**	**15**	**16**	**17**	**18**
19	**20**	**21**	**22**	**23**	**24**
25	**26**	**27**	**28**	**29**	**30**
31	**32**	**33**	**34**	**35**	**36**
37	**38**	**39**	**40**	**41**	**42**
43	**44**	**45**	**46**	**47**	**48**
49	**50**	**51**	**52**	**53**	**54**
55	**56**	**57**	**58**	**59**	**60**
61	**62**	**63**	**64**		

(4) Find the smallest remaining number > 3. It is 5. It has not been crossed out. Circle it and cross out all numbers > 5 which are divisible by seven (every fifth number within columns, and every fifth diagonal; green arrows).

1	2	**3**	**4**	**5**	**6**
7	**8**	**9**	**10**	**11**	**12**
13	**14**	**15**	**16**	**17**	**18**
19	**20**	**21**	**22**	**23**	**24**
25	**26**	**27**	**28**	**29**	**30**
31	**32**	**33**	**34**	**35**	**36**
37	**38**	**39**	**40**	**41**	**42**
43	**44**	**45**	**46**	**47**	**48**
49	**50**	**51**	**52**	**53**	**54**
55	**56**	**57**	**58**	**59**	**60**
61	**62**	**63**	**64**		

(5) Find the smallest remaining number > 6. It is 7. It has not been crossed out. Circle it and cross out all numbers > 7 which are divisible by seven (every seventh number within columns, and every seventh diagonal; purple arrows).

1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
25	26	27	28	29	30
31	32	33	34	35	36
37	38	39	40	41	42
43	44	45	46	47	48
49	50	51	52	53	54
55	56	57	58	59	60
61	62	63	64		

Continue this process until you have crossed out all numbers which are multiples of primes. If the procedure is continued up to *n*, the number of cross-outs gives the number of distinct prime factors of each number. [*CRC Concise Encyclopedia of Mathematics*, Eric W. Weisstein (1999)]

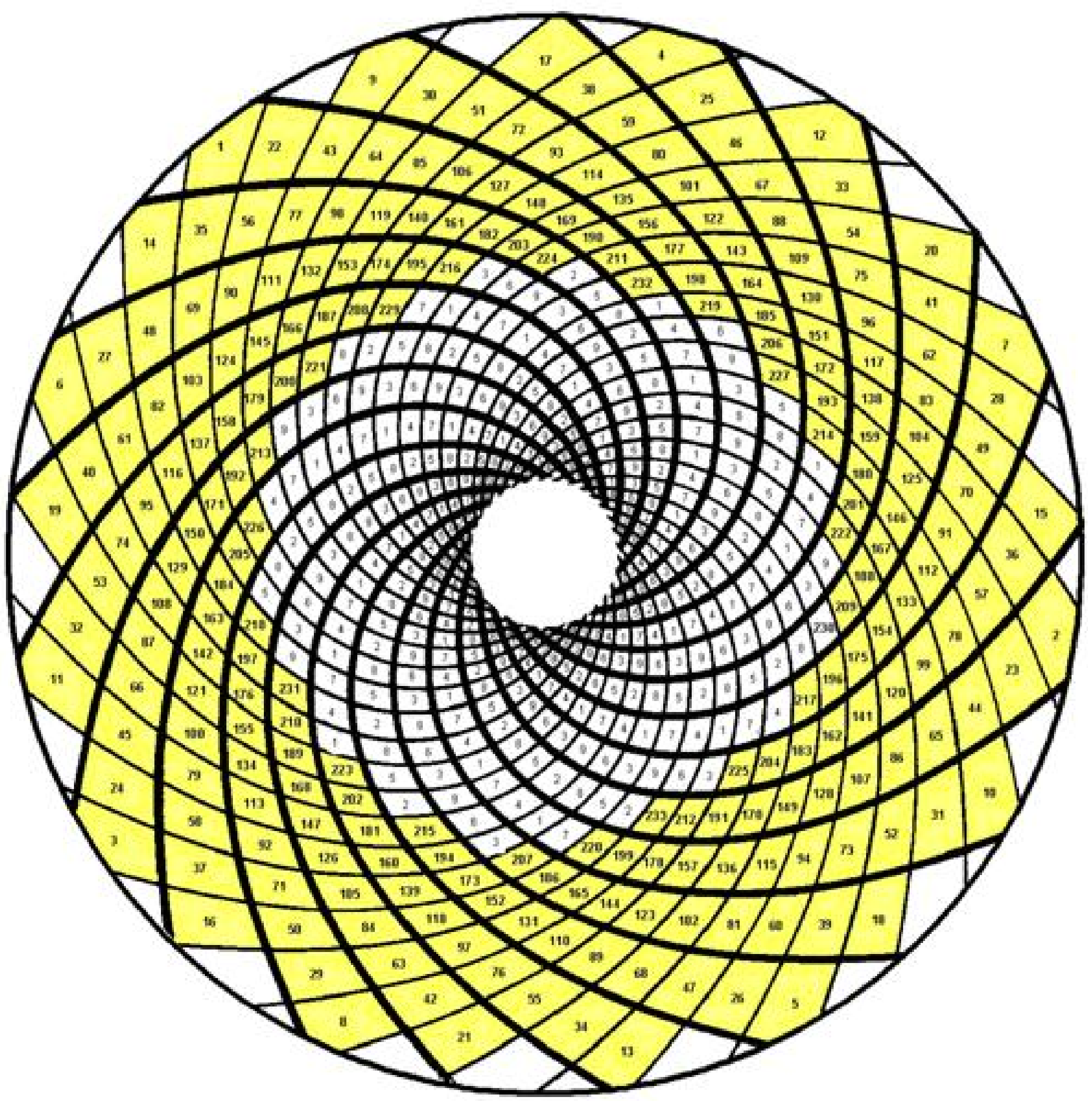
The daisy prime number sieve in 3 dimensions:
A DAISY INFLORESCENCE MODEL (*in situ*).

Our objective in this example is to find the prime numbers less than or equal to 233, using a sieve created from a daisy integer lattice. The number 223 was selected arbitrarily on the basis of the published daisy integer map by Conway and Guy in *The Book of Numbers*.

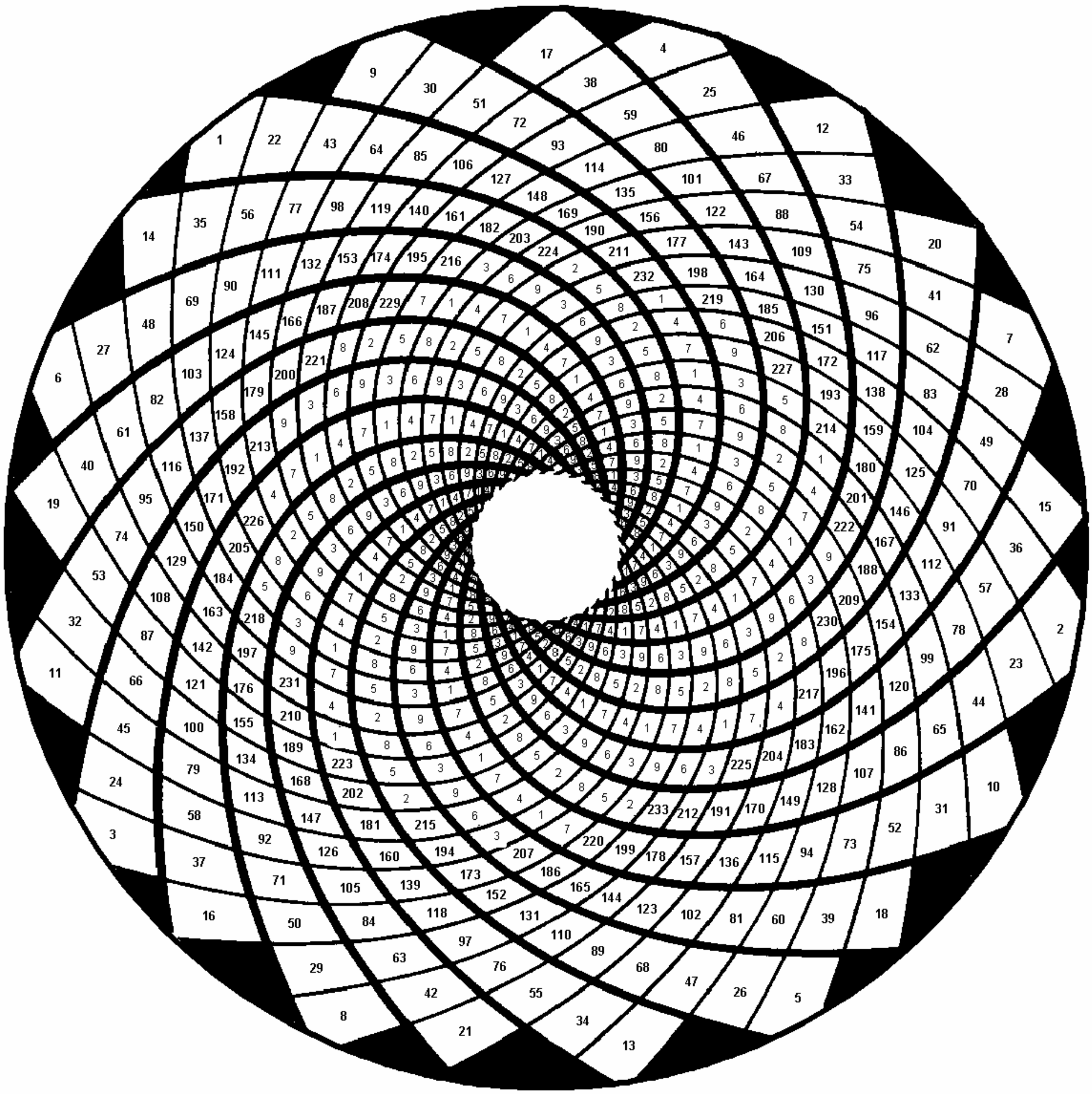
To accomplish this, one must identify the *first 7 primes* in the daisy lattice and then cross out their multiples, leaving only prime numbers behind. In the daisy integer map, the multiples of primes exist conveniently as strings of integers within parastichies. For the first 7 prime numbers these integer strings originate from the lattice node position that is adjacent to the prime number in question, and propagate systematically throughout the lattice.

The first 233 florets (yellow) in the daisy integer lattice

Modified and expanded, after Conway and Guy (1996)



A daisy integer matrix for the first n = 233 florets (numbered 1 to 233).
Florets nearer center are younger than 233, and numbered in modulo 9.



The daisy sieve uses an approach similar to the sieve of Eratosthenes, but the integer matrix is different; making it possible for multiples of primes to be placed in predetermined positions within in spiral sequences within the matrix.

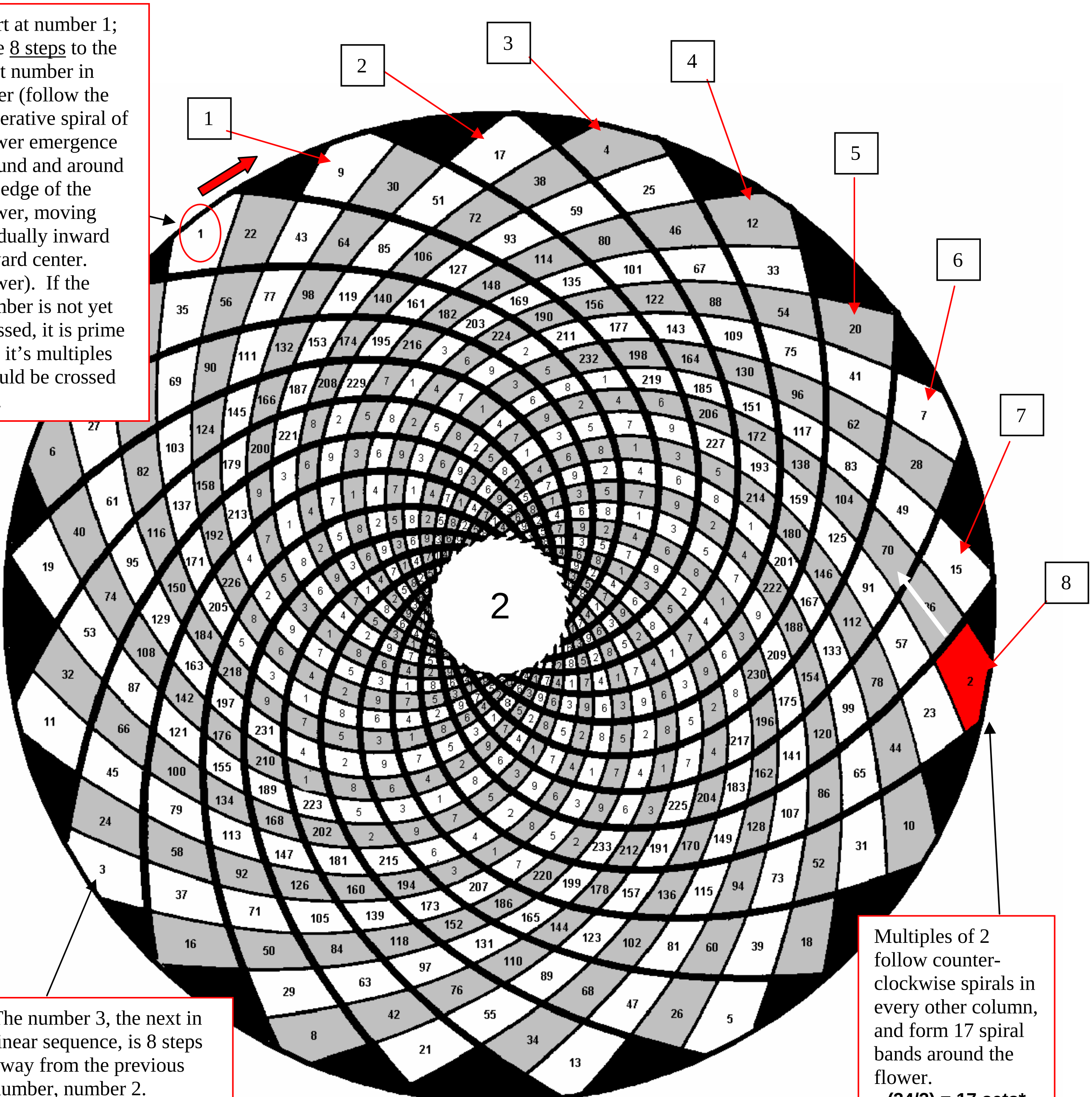
<u>An outline of the daisy sieve algorithm</u>: Starting with '1' and each successive number, move 8 steps along the generative spiral (clockwise) to the next number in sequence from '1' to '233'. If the number has not been crossed out in a previous step, then it is a prime number. Circle it or shade it and then cross out the multiples of that prime; the multiples of primes always lie in unique symmetrical and spiral patterns within the matrix (i.e., as spirals on the surface of a cylinder or cone). The multiples of primes always follow multiplication series that are defined by Fibonacci differences between and among elements (florets) of the matrix. The process of moving from 1 to n produces a spiral that starts from outside and moves to inside, relative the center. (Please refer to following page).

<u>STEP 1</u>. *Locate the first prime number (2) and cross out the multiples of 2.* The first available number in the set of 1 to 233 is 2, at the circumference of the botanical integer lattice. The n umber two is shaded **red**. In a spreadsheet, the number 2 is 13 columns and 8 rows away from the preceding number in the arithmetic sequence (1), as are all consecutive numbers.

In the simulated daisy flower below, the number 2 is 8 steps away from number 1 going clockwise around the matrix. The number 2 has not been crossed out yet, so therefore it is the first prime number.

Then, cross out all multiples of 2. The multiples of 2 occur within every 2nd iteration of the F_{34} parastichy (multiples of 2 are shaded **light gray**).

Start at number 1; take <u>8 steps</u> to the next number in order (follow the generative spiral of flower emergence around and around the edge of the flower, moving gradually inward toward center. flower). If the number is not yet crossed, it is prime and it's multiples should be crossed out.



The number 3, the next in linear sequence, is 8 steps away from the previous number, number 2. Number 3 was not crossed out by the multiples of 2 (light gray shading) yet, so the number 3 also will be prime in the next step.

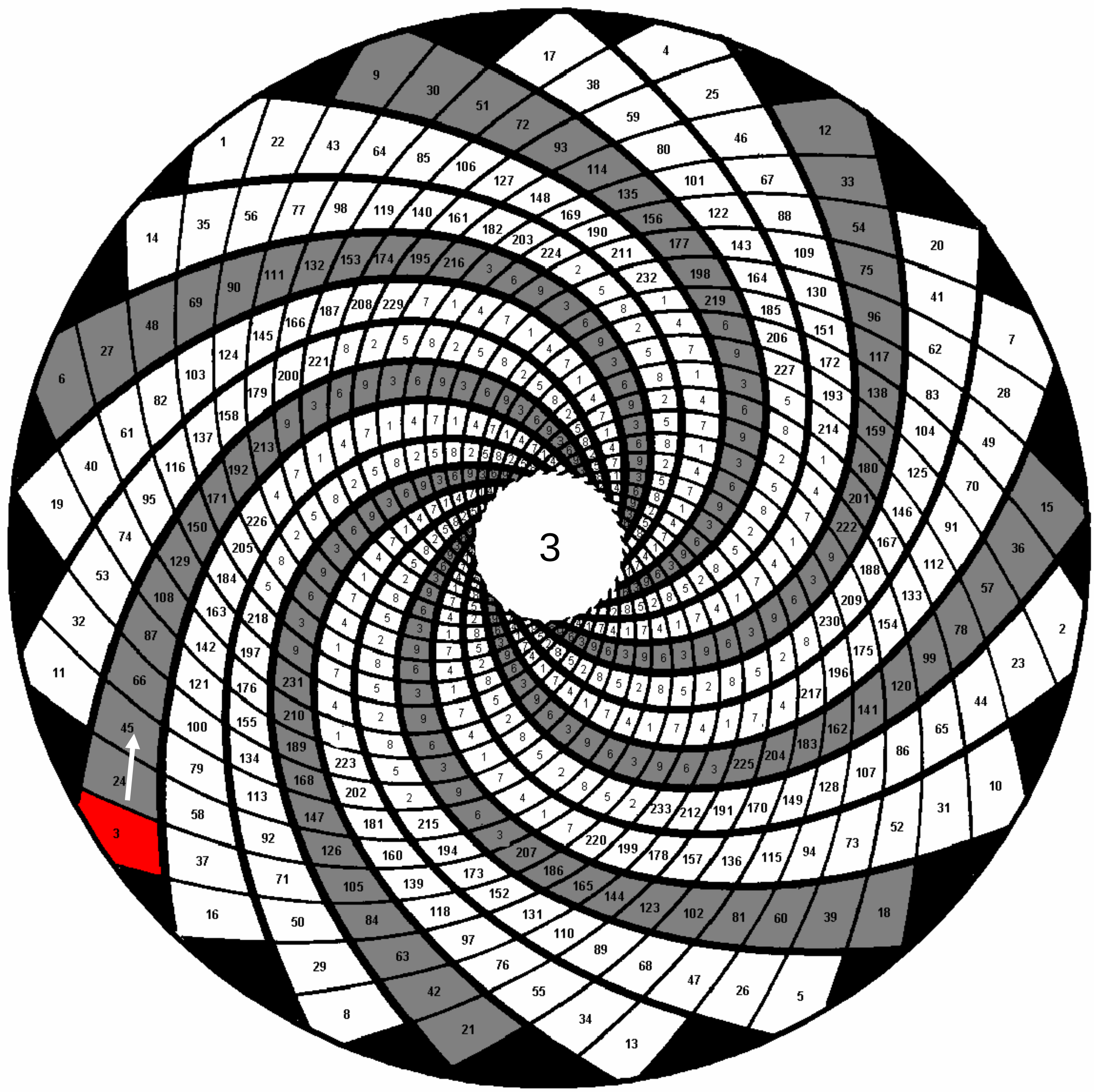
The <u>first</u> prime number , '2' (**red**). Multiples of 2 (**light gray**) occur in every 2nd iteration of the F_{34} parastichy.

Multiples of 2 follow counter-clockwise spirals in every other column, and form 17 spiral bands around the flower.

(34/2) = 17 sets*

(*Formula to determine number of "sets" of multiples of 2 in the matrix.)

<u>STEP 2</u>. *Locate the second prime number (3) and cross out the multiples of 3.* The next available number is 3 (13 columns and 8 rows away from 2 along the generative spiral). It has not been crossed out yet, and therefore 3 is the second prime number. Cross out all multiples of 3. The multiples of 3 occur within every 3rd iteration of the F_{21} parastichy (multiples of 3 are shaded **dark gray**).



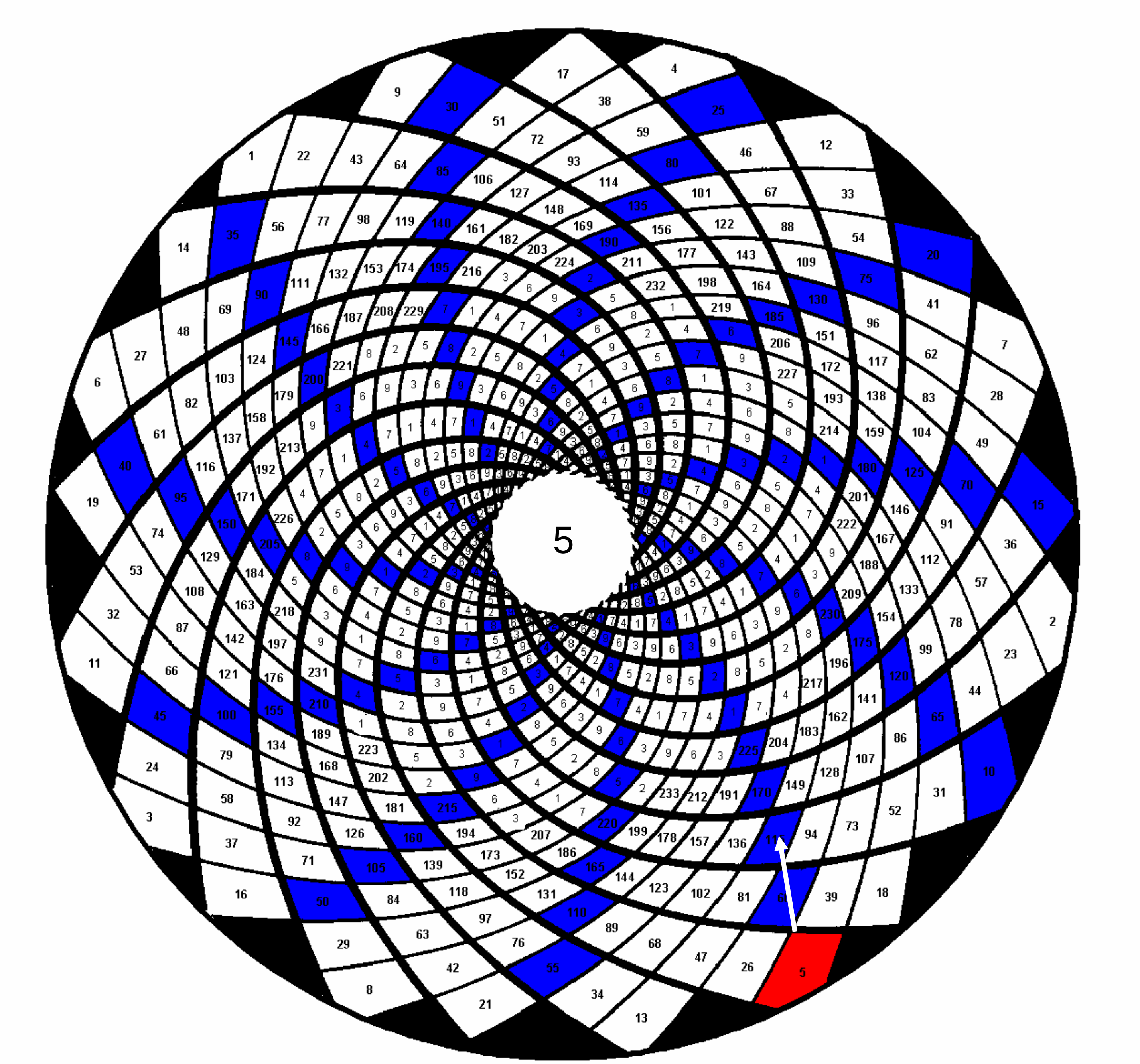
The second prime number, '3' (**red**). Multiples of 2 (shaded dark **gray**) occur in every 3^{nd} iteration of the F_{21} parastichy. Multiples of 3 follow clockwise spirals in every third row, forming 7 spiral bands around the flower. The white arrow shows the direction in which multiples of primes are crossed out along the 7 spiral bands.

Formula to determine number of "sets" of multiples of 3 in the matrix:

$$(21/3) = 7 \text{ sets}$$

where 21 = the number of clockwise spirals, and 3 = the prime number.

<u>STEP 3</u>. *Locate the third prime number (5) and cross out the multiples of 5.* The next available number is 5 (13 columns and 8 rows away from 4 along the generative spiral). It has not been crossed out, and therefore 5 is the third prime number. Cross out all multiples of 5, i.e., integers after 5 within every 5th iteration of the F_{55} parastichy (multiples of 5 are shaded **dark blue**).



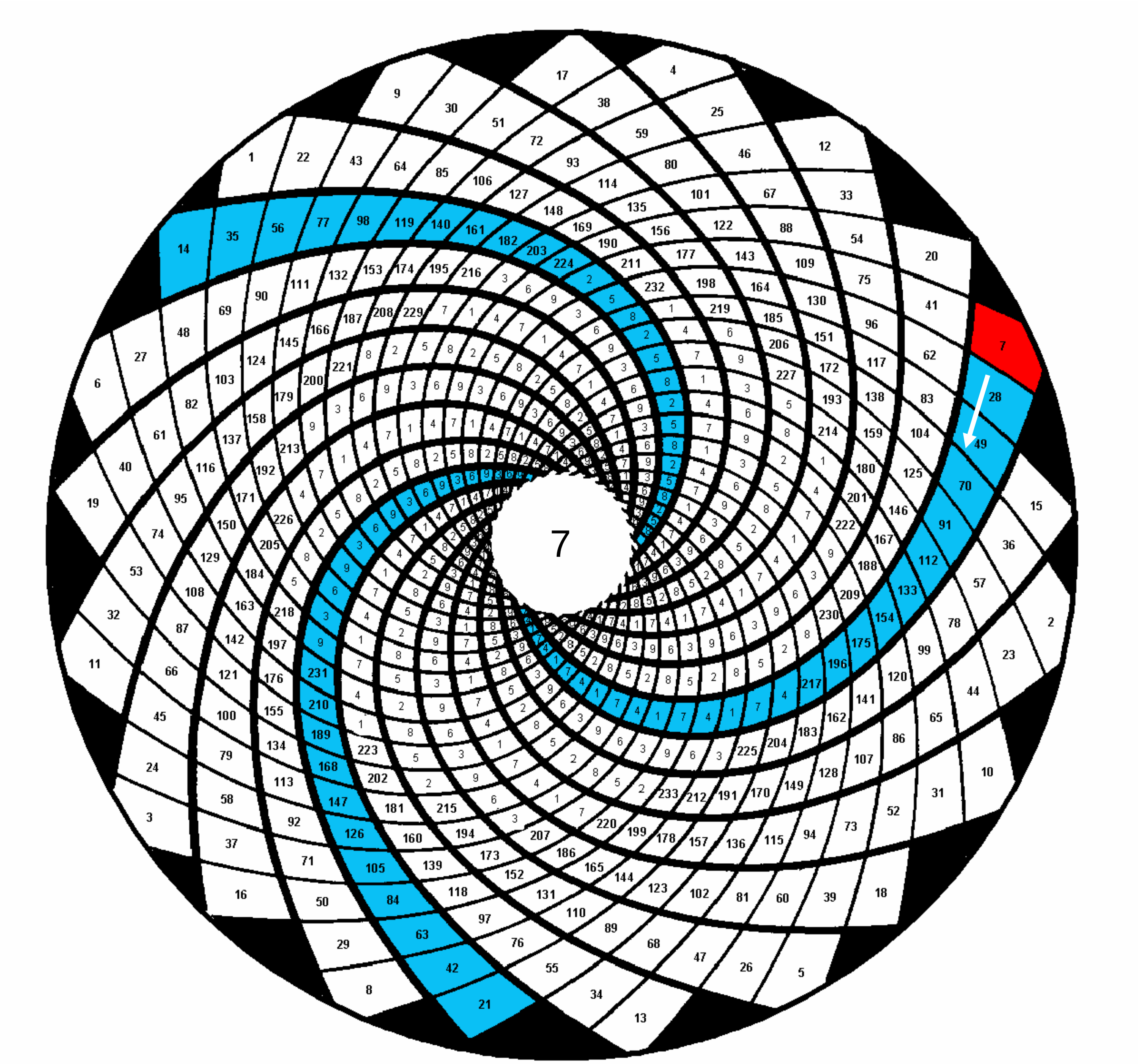
Multiples of 5 (**dark blue**) occur in every 5th iteration of the F_{55} parastichy. Multiples of 5 follow counter-clockwise spirals in every 5th diagonal of the matrix, forming 11 spiral bands around the flower (determined by formula below). The white arrow shows the direction in which multiples of primes are crossed out along the 11 spiral bands.

Formula to determine number of "sets" of multiples of 5 in the matrix:

$$(21 + 34)/5 = 11 \text{ sets}$$

where 21 and 34 are the Fibonacci number of opposing spirals in the daisy matrix, and 5 is the prime number.

<u>STEP 4</u>. *Locate the fourth prime number (7) and cross out the multiples of 7.* The number 6 was already crossed out in a previous step. The next potential prime number is the number 7 (13 columns and 8 rows from 6 along the generative spiral). The number 7 has not been crossed out yet, and therefore 7 is the fourth prime number. Cross out all multiples of 7, i.e., integers within every 7^{th} iteration of the F_{21} parastichy (multiples of 7 are shaded **light blue**).



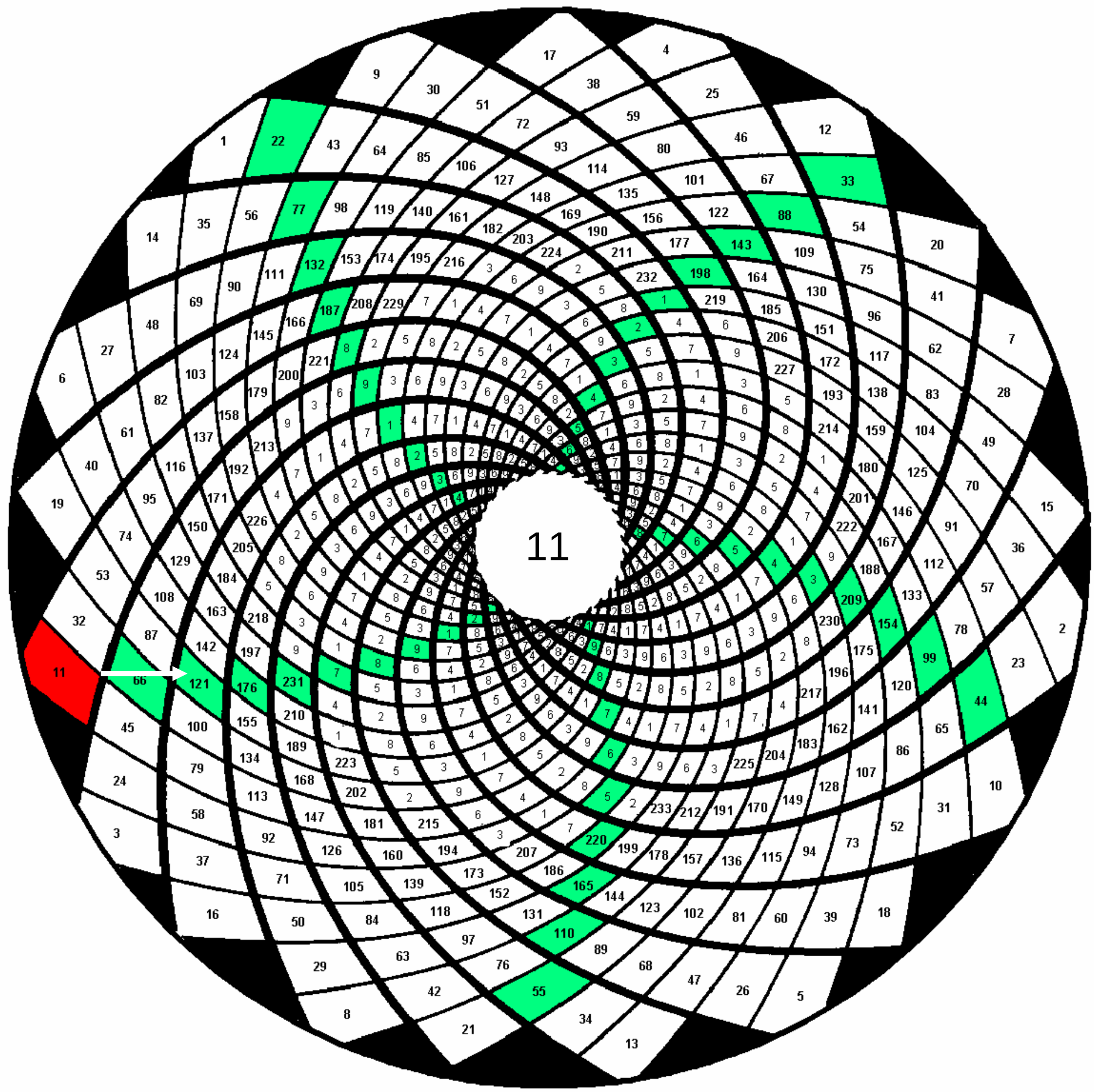
Multiples of 7 (**light blue**) occur in every 7^{th} iteration of the F_{21} parastichy. Multiples of 7 follow clockwise spirals in every 7^{th} row of the matrix, forming 3 spiral bands around the flower (determined by formula below). The white arrow shows the direction in which multiples of primes are crossed out along the 3 spiral bands.

Formula to determine number of “sets” of multiples of 7 in the matrix:

(21/7) = 3 sets

where 21 is number of one set of opposing spirals in the daisy matrix, and 7 is the prime number.

<u>STEP 5</u>. *Locate the fifth prime number (11) and cross out the multiples of 11.* The numbers 8, 9, and 10 were already crossed out in a previous step. The next potential prime number is the number 11 (13 columns and 8 rows from 10 along the generative spiral). The number 11 has not been crossed out yet, and therefore 11 is the fifth prime number. Cross out all multiples of 11, i.e., integers within every 11th iteration of the F_{55} parastichy (multiples of 11 are shaded **light green**).



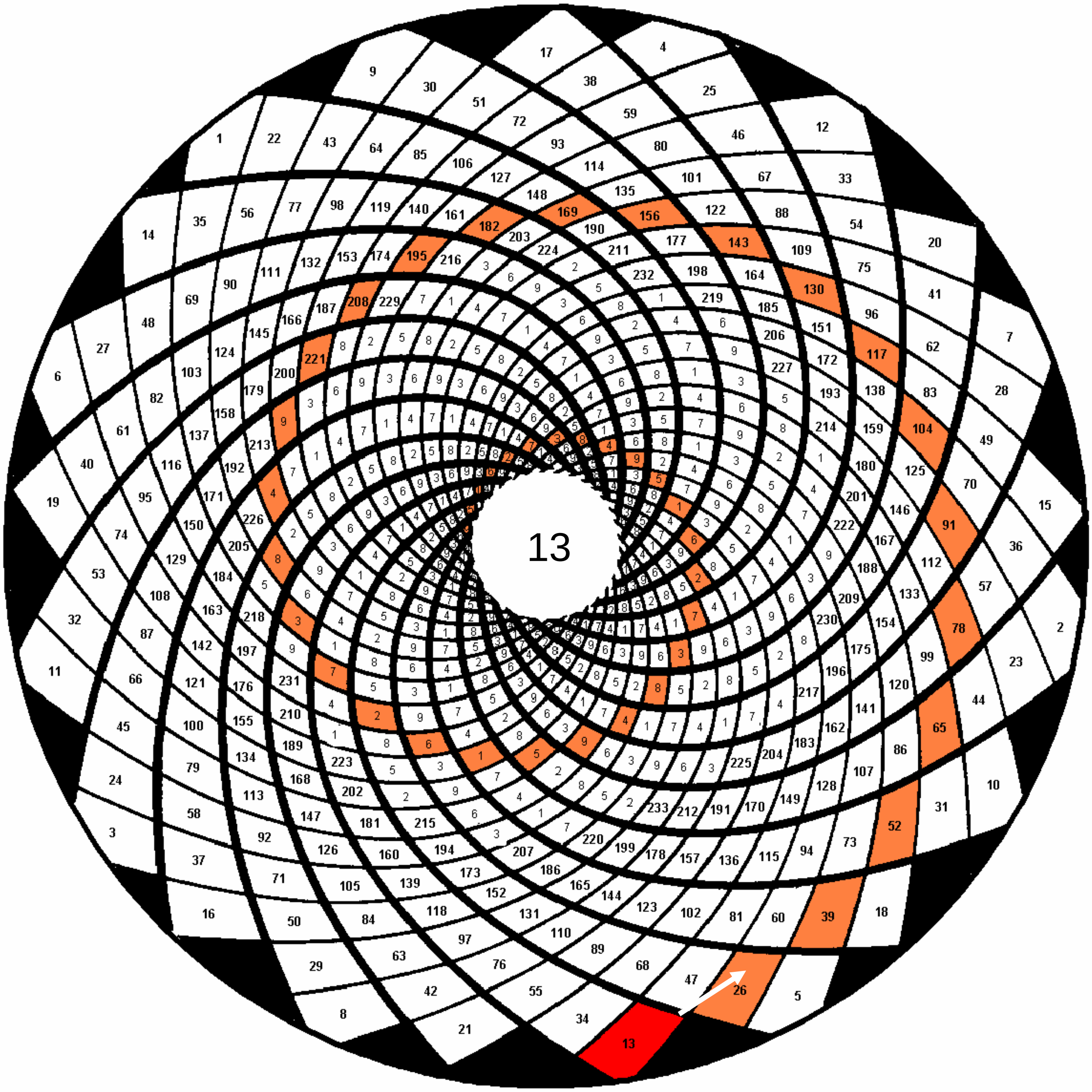
Multiples of 11 (**light green**) occur in every 5th iteration of the F_{55} parastichy. Multiples of 11 follow counter-clockwise spirals in every 11th diagonal of the matrix, forming 5 spiral bands around the flower (determined by formula below). The white arrow shows the direction in which multiples of primes are crossed out along the 11 spiral bands.

Formula to determine number of "sets" of multiples of 13 in the matrix.

$$(21 + 34)/11 = 5 \text{ sets}$$

where 21 and 34 are the Fibonacci number of opposing spirals in the daisy matrix, and 11 is the prime number.

<u>STEP 6</u>. *Locate the sixth prime number (13) and cross out the multiples of 13.* The number 12 was already crossed out in a previous step. The next potential prime number is the number 13 (13 columns and 8 rows from 6 along the generative spiral). The number 13 has not been crossed out yet, and therefore 13 is the sixth prime number. Cross out all multiples of 13, i.e., integers within every 13th iteration of the F_{13} parastichy (Fig. 7, multiples of 13 are shaded **orange**).



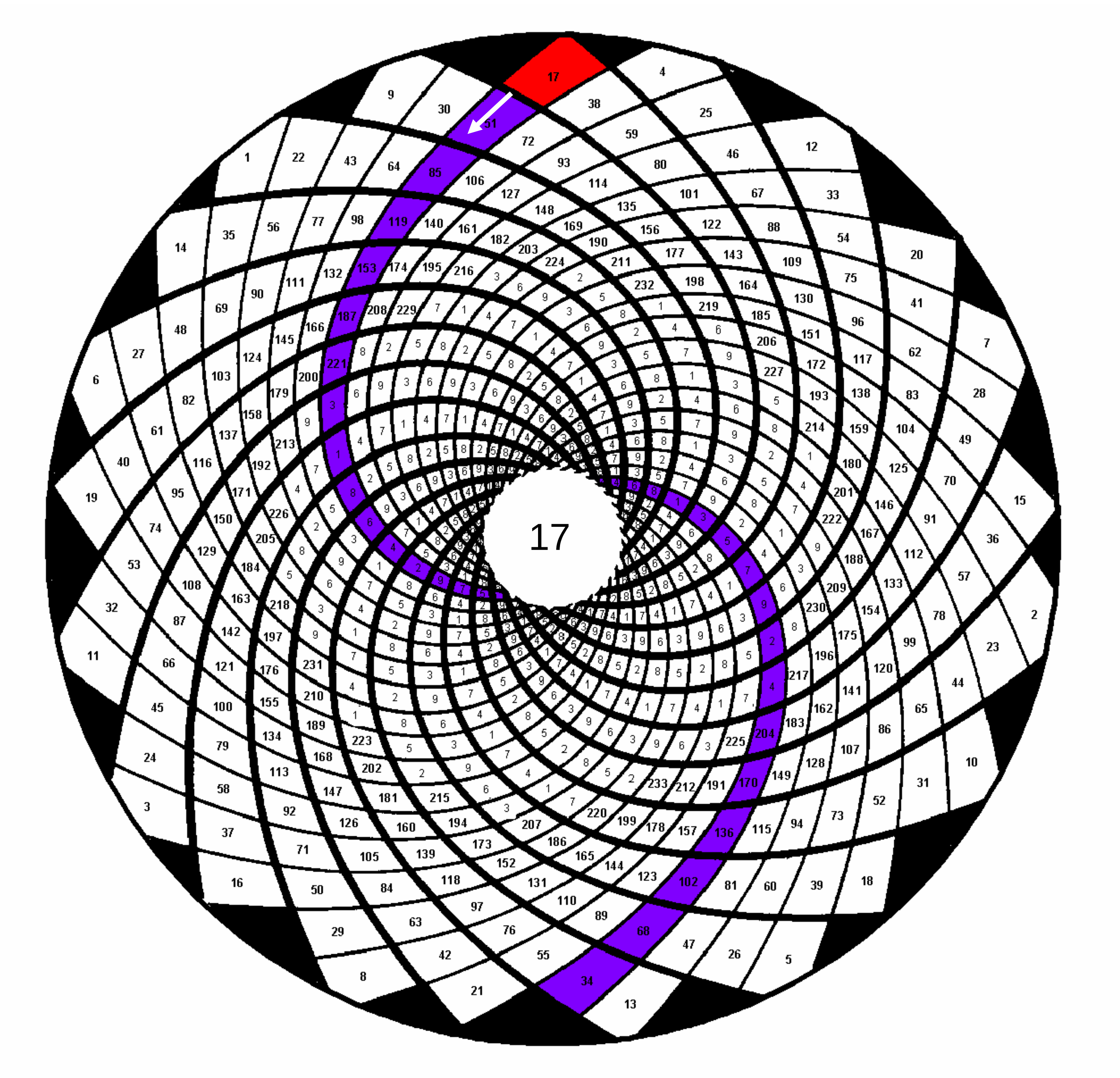
Multiples of 13 (**orange**) occur in every 13th iteration of the F_{13} parastichy. Multiples of 13 form a single, counter-clockwise spiral in the matrix. The numbers in sequence are determined by ($n_2 = n_1 + 13$) … ,etc. There are 13 possible spirals similar to the one occupied by the multiples of 13. The 13 spirals originate with the numbers [13, 5, 10, 2, 7, 12, 4, 9, 1, 6, 11, 3, 8].

Formula to determine number of "sets" of multiples of 13 in the matrix.

13/13 = 1 set

where 13 (numerator) = number of spirals and 13 (denominator) = the prime number.

<u>STEP 7</u>. *Locate the seventh prime number (17) and cross out the multiples of 17.* The numbers 14, 15, and 16 were already crossed out in a previous step. The next potential prime number is the number 17 (13 columns and 8 rows from 10 along the generative spiral). The number 17 has not been crossed out yet, and therefore 17 is the seventh prime number. Cross out all multiples of 17, i.e., integers within every 17th iteration of the F_{34} parastichy (Fig. 7, multiples of 17 are shaded **purple**).



Multiples of 17 (**purple**) occur in every 17th iteration of the F_{34} parastichy. (34/2) = 17 sets*

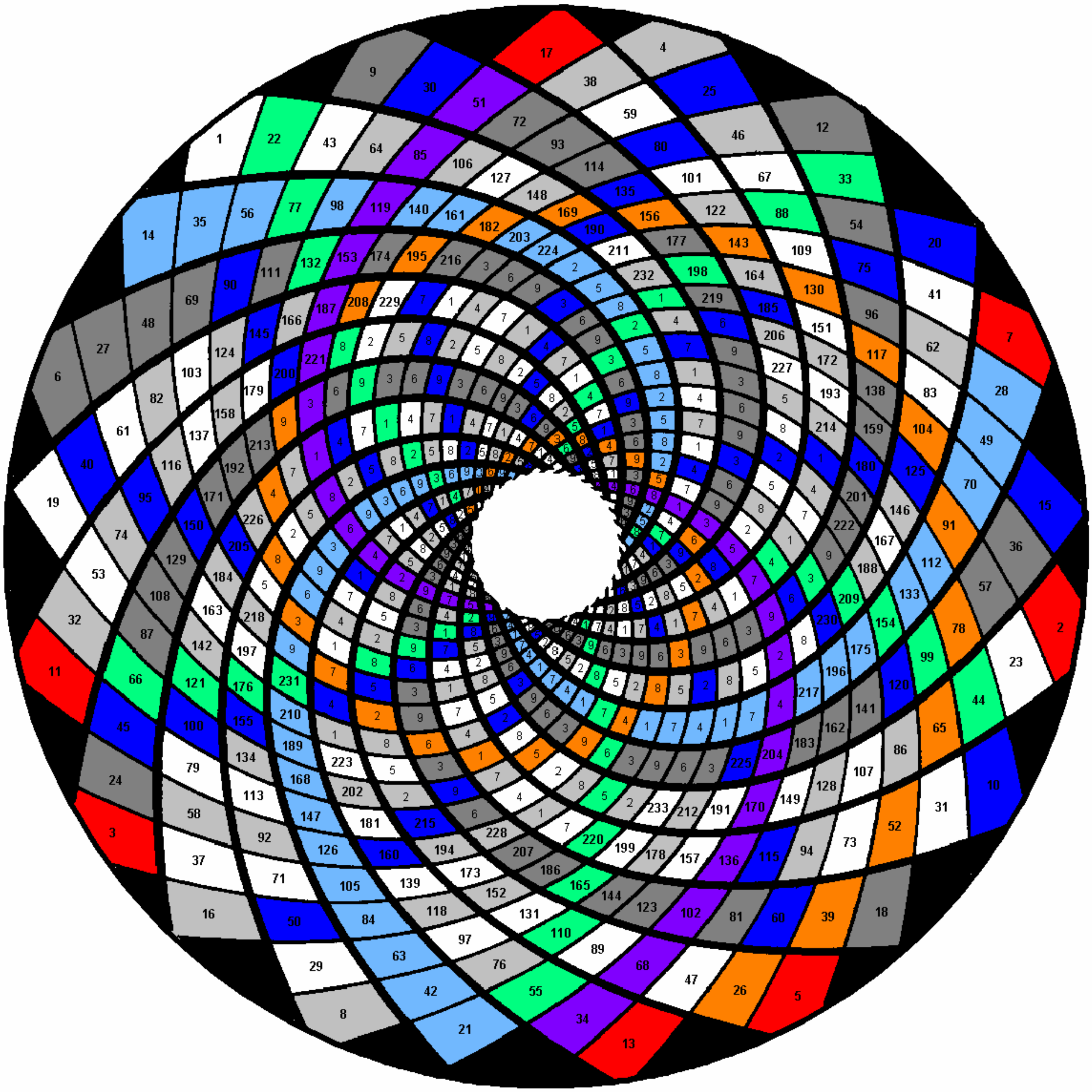
Formula to determine number of “sets” of multiples of 2 in the matrix:

(34/17) = 2 sets

where 34 = the number of counter-clockwise spirals in the matrix and 17 is the prime number.

The completed daisy phyllotaxis prime sieve for the first n=233 florets. This completes the sieving algorithm for primes for n = 233. All remaining numbers in the lattice that are less than or equal to 233 and have no shading are the *prime numbers*. To sieve larger prime numbers, expand the matrix and cross out multiples of primes along 2nd order or higher order parastichies. The number of cross-outs (color overlaps) gives the number of distinct prime factors of each number.

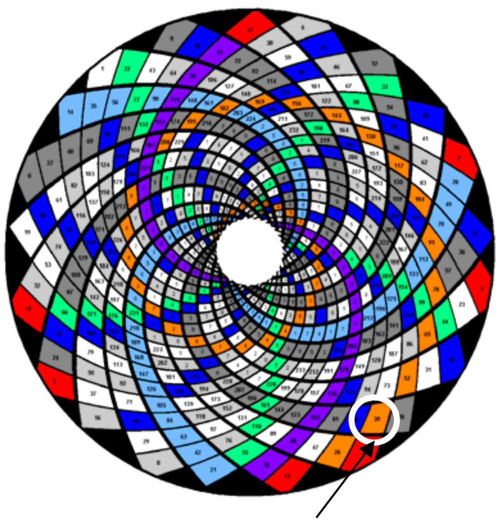
CONCLUSION: the multiples of each prime number follow unique spiral paths along the surface of the flower form simple multiplication series associated with Fibonacci differences.



The un-shaded numbers in the matrix up to n = 233 are primes. The un-shaded numbers in the center of the matrix (in modulo 9) are *potential* primes to be determined by higher order sieving.

PRIME FACTORS.

Similar to Sieve of Eratosthenes, the number of "cross-outs" in the daisy matrix gives the number of distinct *prime factors* of each number.



For example, the circled number (39) was crossed out twice:
first as multiples of 3 (**dark gray** shading) and then as multiples of 13 (**orange** shading).
Therefore, these two "cross-outs" indicate that the number 39 has two prime factors,
the numbers 3 and 13.

ULAM SPIRAL COMPARISON. It would be interesting so observe the patterns of primes in a much larger configuration and compare the organic form (daisy) with the arbitrary form posed by Ulam.